



Flow and convective heat transfer characteristics of water-based Al_2O_3 nanofluids in fully developed laminar flow regime

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ABSTRACT

We have measured the pressure drop and convective heat transfer coefficient of water-based Al_2O_3 nanofluids flowing through a uniformly heated circular tube in the fully developed laminar flow regime. The experimental results show that the data for nanofluid friction factor show a good agreement with analytical predictions from the Darcy's equation for single-phase flow. However, the convective heat transfer coefficient of the nanofluids increases by up to 8% at a concentration of 0.3 vol% compared with that of pure water and this enhancement cannot be predicted by the Shah equation. Furthermore, the experimental results show that the convective heat transfer coefficient enhancement exceeds, by a large margin, the thermal conductivity enhancement. Therefore, we have discussed the various effects of thermal conductivities under static and dynamic conditions, energy transfer by nanoparticle dispersion, nanoparticle migration due to viscosity gradient, non-uniform shear rate, Brownian diffusion and thermophoresis on the remarkable enhancement of the convective heat transfer coefficient of nanofluids. Based on scale analysis and numerical solutions, we have shown, for the first time, the flattening of velocity profile, induced from large gradients in bulk properties such as nanoparticle concentration, thermal conductivity and viscosity. We propose that this flattening of velocity profile is a possible mechanism for the convective heat transfer coefficient enhancement exceeding the thermal conductivity enhancement.

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1. Introduction

A liquid coolant is widely used to prevent the overheating or heat transfer rate improvement of equipments such as electronic devices, heat exchangers and transportation vehicles. However, conventional heat transfer fluid such as water or ethylene glycol generally has poor thermal properties. So, many efforts for dispersing small particles with high thermal conductivity in the liquid coolant have been conducted to enhance thermal properties of the conventional heat transfer fluids [1–23]. The early research, which used suspension and dispersion of millimeter- and micrometer-sized particles, faced the major problem of poor suspension stability. Thus, a new class of fluid for improving both thermal conductivity and suspension stability is required in the various industrial fields. This motivation leads to development of nanofluids. Nanofluid is a new kind of fluid consisting of uniformly dispersed and suspended nanometer-sized particles or fibers in fluids and has unprecedented thermal characteristics [1–21]. They were first pioneered by Choi [1] in 1995. Since his work [1] many researchers have discovered the attractive features of nanofluids such as the anomalously high thermal conductivity at very low nanoparticles

concentration [1–8] and the considerable enhancement of forced convective heat transfer [12–20]. One of particular interests in convective heat transfer of nanofluids is that the increment of convective heat transfer coefficient is generally higher than that of effective thermal conductivity [12–14,16] and moreover the exact mechanism of convective heat transfer enhancement of nanofluids has not been explained in detail, although it becomes well known that the enhancement of thermal conductivity of nanofluids is due to Brownian motion of nanoparticles suspended in fluid [6–8].

Previous investigations on the convective heat transfer enhancement of nanofluids have been reported as follows: Xuan and Li [13] studied the single-phase flow and heat transfer performance of nanofluids under turbulent flow in tubes. Their experimental results showed that the convective heat transfer coefficient and the Nusselt number of nanofluids increase with the Reynolds number and the volume fraction of nanoparticles under turbulent flow. Compared with water, the Nusselt number of the nanofluids with a 2.0 vol% of Cu nanoparticles is more increased than 39%. Wen and Ding [14] focused on the entry region under laminar flow condition using nanofluids containing $\gamma\text{-Al}_2\text{O}_3$ nanoparticles of various concentrations. They presented that the local heat transfer coefficient at axial positions in the entry region is about 41% and 47% higher at $Re = 1050$ and 1600 in comparison with water, respectively, and the enhancement just decreases

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Nomenclature

A_c	cross-sectional area of tube (m^2)	Pr	Prandtl number
B	bias error	q''	heat flux (W/m^2)
C_p	heat capacity ($\text{kJ}/\text{kg K}$)	R	precision error
D_B	Brownian diffusion coefficient (m^2/s)	Re	Reynolds number
D_T	thermal diffusion coefficient (m^2/s)	S_F	standard deviation
d_p	particle diameter (m)	T_m	mean temperature ($^\circ\text{C}$)
d_{tube}	inner diameter of a tube (m)	T_s	wall temperature of a tube ($^\circ\text{C}$)
f	friction factor	U	measurement uncertainty
h	convective heat transfer coefficient ($\text{W}/\text{m}^2 \text{K}$)	u	fluid velocity (m/s)
k	thermal conductivity ($\text{W}/\text{m K}$)	X	measurement value
k_B	Boltzmann constant (J/K)		
K	thermophoretic coefficient		
K_μ	proportionality constant related to the viscosity gradient	<i>Greek symbols</i>	
K_c	proportionality constant related to a non-uniform shear rate	α	thermal diffusivity (m^2/s)
L	length of a tube (m)	ϕ	volume fraction
\dot{m}	mass flow rate (kg/s)	$\dot{\gamma}$	shear rate (1/s)
N	number of data	λ	degree of freedom
Nu	Nusselt number	μ	viscosity ($\text{N s}/\text{m}^2$)
P	surface perimeter (m)	ρ	density (kg/m^3)
ΔP	pressure drop (N/m^2)	<i>Subscripts</i>	
		Nano	nanofluids
		BF	base fluid

along the axial direction. It is shown that the enhancement increases with the Reynolds number as well as the volume concentration of nanoparticle. Yang et al. [15] investigated convective heat transfer coefficient of nanofluids for a constant temperature under laminar flow in a horizontal tube heat exchanger. Their experimental results presented that the nanofluids with 2.5 wt% nanoparticles loading had a typical increase in convective heat transfer coefficient of 22% over the base fluid, while its thermal conductivity was about 50% higher than that of the base fluid. This is a different result from that presented by previous papers [12–14,16].

As shown in the previous results [12–17], these papers do not consider fully developed laminar flow, but turbulent flow or entry region of laminar flow. However, study on fully developed laminar flow is more important for understanding the physical phenomena than any other regime because the boundary layer does not grow any longer, and velocity and dimensionless temperature profiles do not change with axial distance under fully developed laminar flow. Therefore, in this paper, our goal is to investigate flow and convective heat transfer characteristics of water-based Al_2O_3 nanofluids flowing through a circular tube with the constant heat flux in fully developed laminar regime. Also, based on the experimental results, the various effects of thermal conductivities under static and dynamic conditions, energy transfer by nanoparticle dispersion, particle migration due to viscosity gradient, non-uniform shear rate, Brownian diffusion and thermophoresis on the remarkable enhancement of the convective heat transfer coefficient are discussed to understand convective heat transfer characteristics of water-based Al_2O_3 nanofluids flowing through a circular tube.

2. Experimental system and validation

Fig. 1 shows experimental apparatus for investigation on flow and convective heat transfer characteristics of water-based Al_2O_3 nanofluids flowing through a circular tube with the constant heat flux in fully developed laminar flow regime. It mainly consists of a test section, a pump, a reservoir tank, and a cooler. The straight stainless steel tube with 1.812 mm inner diameter and 2500 mm length is used as the test section. The tube surface is electrically

heated by an AC power supply to generate constant heat flux ($q'' = 5000 \text{ W}/\text{m}^2$) and is insulated thermally by about 150 mm-thick blanket to minimize the heat loss from the tube to the ambient. To measure the wall temperature of the stainless steel tube and the mean temperature of the fluids at the inlet, five thermocouples (T-type) are soldered on at different places along the test section and one thermocouple (T-type) is inserted at the inlet of the test section. The pressure drop is measured by a pressure transducer (DP 10-34) at both ends of test section. A pump (HV-77921-40) controls the flow rate of the fluid in a range from 0.4 to 21 ml/min. To preserve a constant temperature at the inlet of the test section the heated fluid returns to the reservoir tank passing through a cooler.

In this experiment, water-based Al_2O_3 nanofluids with various volume fractions ranging from 0.01% to 0.3% are manufactured by the two-step method which indicates that base fluid including nanoparticles is sonicated in an ultrasonic bath. The size of nanoparticles made by Nanotechnologies is $30 \pm 5 \text{ nm}$. Zeta potential of water-based Al_2O_3 nanofluids used in this work is larger than about 30 mV. The stability of water-based Al_2O_3 nanofluids is physically stable [21].

All physical properties of the water-based Al_2O_3 nanofluids needed to calculate the pressure drop and the convective heat transfer coefficient are measured as follows: the heat capacity (C_p) is measured using DSC 204 F1 manufactured by NETZSCH, the density (ρ) is measured using a BX 300 manufactured by Shimadzu, the dynamic viscosity (μ) is measured using a VM-10-A manufactured by Viscometer and the thermal conductivity (k) is measured by a transient hot wire method. The heat capacity of water-based nanofluids (0.01–0.3 vol%) is measured and the value is similar to that of water. In addition, measured values of the density agrees well with the values calculated from mixing theory [24], as shown in Fig. 2. Fig. 3 shows the dynamic viscosity of water-based Al_2O_3 nanofluids is increased about 2.9% for a 0.3 vol% at 21°C compared with that of pure water and presents a similar tendency to the results presented by Wang et al. [4]. However, the Einstein's model [25] fails to predict the dynamic viscosity measurement of water-based Al_2O_3 nanofluids [21]. Also, Fig. 3 shows the thermal conductivity of water-based Al_2O_3 nano-

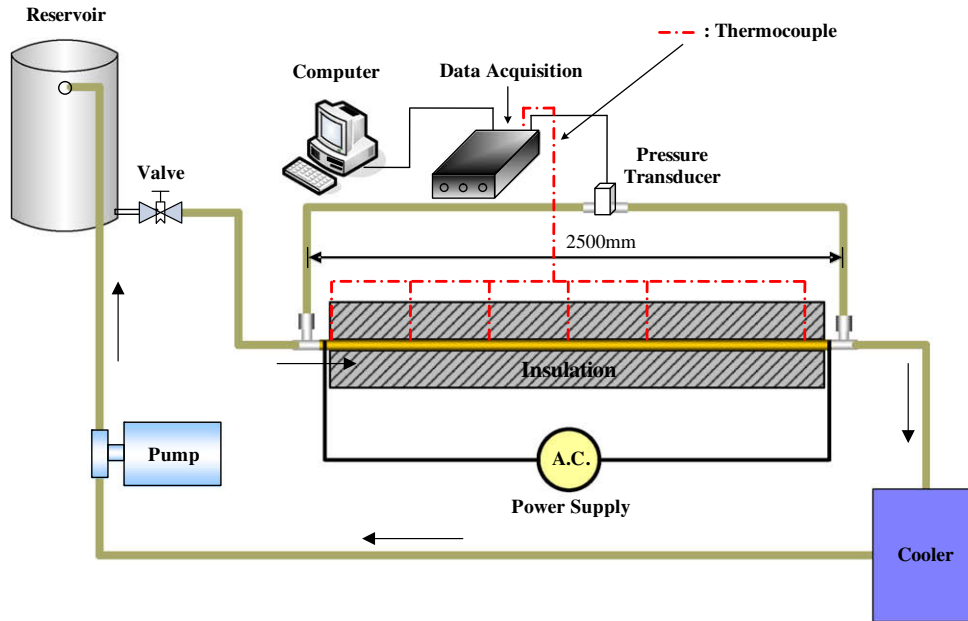


Fig. 1. Schematic diagram of experimental apparatus.

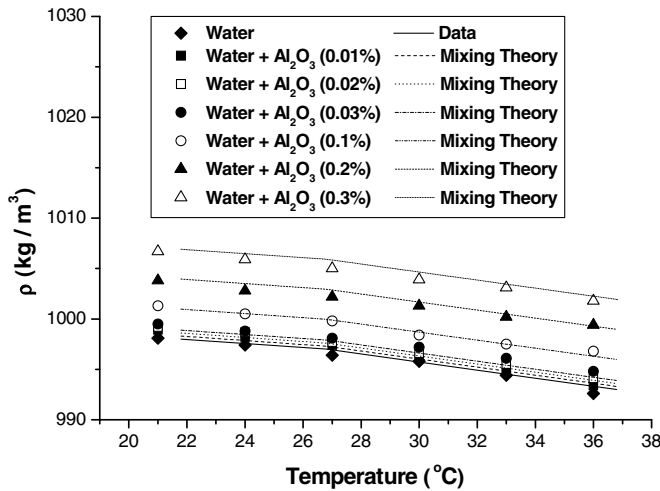


Fig. 2. Density of water-based Al₂O₃ nanofluids.

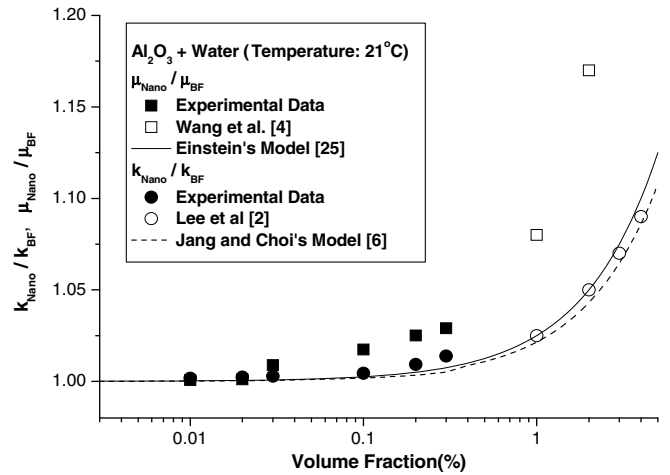


Fig. 3. Dynamic viscosity and thermal conductivity of water-based Al₂O₃ nanofluids.

fluids is enhanced by 1.44% for a 0.3 vol% at 21 °C compared with that of pure water, and Jang and Choi's model [6] can predict both experimental results and the results presented by the Lee et al. [2] within 1% error.

With the measured wall temperature at the stainless steel tube, the mean temperature of fluids at inlet, heat flux, and flow rate, the local heat transfer coefficient of nanofluids under the fully developed condition of laminar flow is calculated by

$$h(x) = \frac{q''}{T_s(x) - T_m(x)} \quad (1)$$

where $T_m(x)$, $T_s(x)$ and $h(x)$ are the mean temperature of fluid, the wall temperature of tube and the heat transfer coefficient, respectively. Also, with the energy balance in a tube the mean temperature of fluid can be given by

$$T_m(x) = T_{m,i} + \frac{q''P}{\dot{m}C_p}x \quad (2)$$

where $T_{m,i}$, P , \dot{m} and C_p are the fluid temperature at the inlet of the test section, the surface perimeter, the mass flow rate and the heat capacity, respectively. Therefore, the local heat transfer coefficient can be obtained with Eqs. (1) and (2).

In order to verify the accuracy and the reliability of this experimental system, the pressure drop and the local heat transfer coefficient are experimentally measured using water before obtaining those of water-based Al₂O₃ nanofluids. The experiments on the pressure drop are conducted within the Reynolds number of 600, and entry length is within 2% of total tube length. Therefore, we can assume the fully developed laminar flow regime. Fig. 4 shows that pressure drop data for water and analytical predictions using Eq. (3) have a good agreement with less than 1% error

$$\Delta P = 32 \frac{\mu_m \mu L}{d_{tube}^2} \quad (3)$$

The heat transfer coefficients calculated by Eqs. (1) and (2) are compared with the following Shah equation [26,27] of the local Nusselt

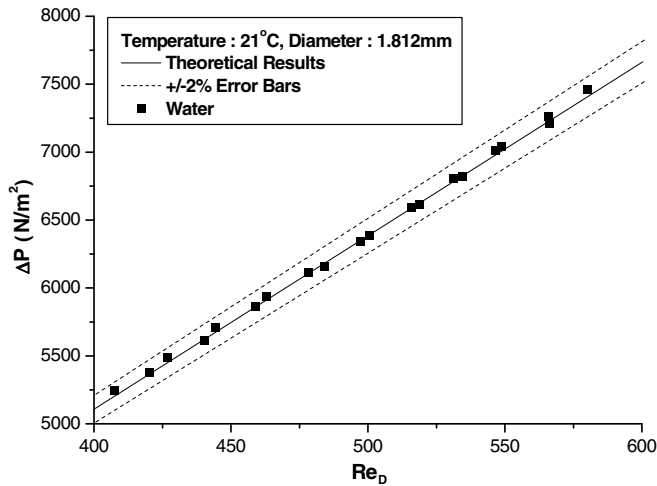


Fig. 4. Comparison between theoretical and experimental pressure drops of water.

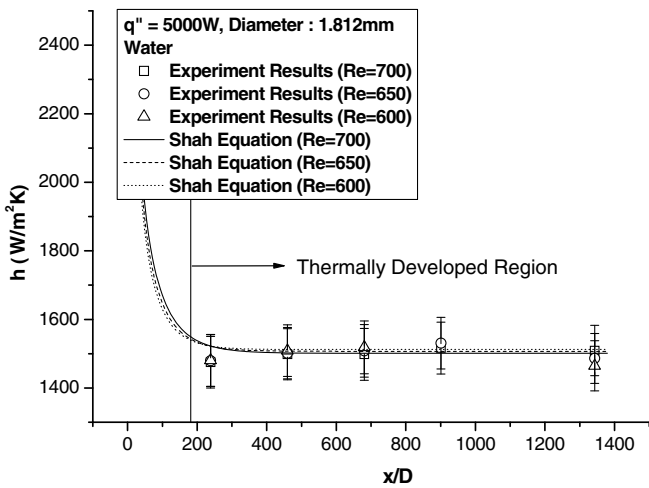


Fig. 5. Comparison between Shah equation and experimental data for the convective heat transfer coefficient of water.

number in a circular tube for a constant heat flux condition according to the Reynolds number in Fig. 5

$$Nu_x = \begin{cases} 1.302x_*^{-1/3} - 1, & x_* \leq 0.00005 \\ 1.302x_*^{-1/3} - 0.5, & 0.00005 \leq x_* \leq 0.0015 \\ 4.364 + 8.68(10^3 x_*)^{-0.506} \exp(-41x_*), & x_* \geq 0.001 \end{cases} \quad (4)$$

where $Nu_x = h(x)d_{tube}/k$, $x_* = [(x/d_{tube})/(Re_{d_{tube}} Pr)]$. For the fully developed laminar flow regime, the experimental data compared with Shah equation [26,27] have a good agreement within 3% error and are constant.

2.1. Uncertainty analysis

In the experiment, the measurement error divided into calibration errors, data acquisition errors, and data reduction errors consists of precision (random) error from the repeatability of the data and bias error from the thermocouple and pressure transducer. The bias error of the data acquisition is negligible because it is very small. The uncertainty evaluation is performed in accordance with a 95% confidence interval

$$U = \sqrt{B^2 + (t_{\lambda,95\%} R)^2} \quad (5)$$

where U , B , and $t_{\lambda,95\%} R$ are measurement uncertainty, bias error, and estimate of the precision error in the repeated temperature and pressure measurement at 95% confidence. In addition, λ is the degree of freedom:

$$\lambda = N - 1 \quad (6)$$

where N is the number of data. The precision error is determined by

$$R = \frac{S_F}{N^{1/2}} \quad (7)$$

$$S_F = \sqrt{\frac{1}{N} \sum_{i=1}^N (X_i - \bar{X})^2} \quad (8)$$

where S_F and $(X_i - \bar{X})$ are standard deviation and deviation of X_i . The true value is given as

$$X' = \bar{X} \pm U(95\%) \quad (9)$$

where X' and \bar{X} are true temperature or pressure and mean temperature or pressure measured.

3. Results and discussion

The pressure drop of water-based Al_2O_3 nanofluids flowing through a circular tube is experimentally measured to investigate flow characteristics of the nanofluids.

Based on the pressure drop of water-based Al_2O_3 nanofluids, we can express the Darcy friction factor, which is a dimensionless parameter defined as

$$f = \frac{-(dp/dx)d_{tube}}{\rho u_m^2/2} \quad (10)$$

For fully developed laminar flow, it follows that

$$f = \frac{64}{Re_{d_{tube}}} \quad (11)$$

Substituting the measured pressure drop into Eq. (10), the Darcy friction factor can be calculated. Fig. 6 shows that the Darcy friction factor of water-based Al_2O_3 nanofluids in fully developed laminar flow regime has a good agreement with Eq. (11) with less than 2.4% error. This implies that the friction factor correlation for the single-phase flow can be extended to water-based Al_2O_3 nanofluids. Interestingly, this trend is in general agreement with the findings of Xuan and Li [13] and Pak and Cho [12] under turbulent flow.

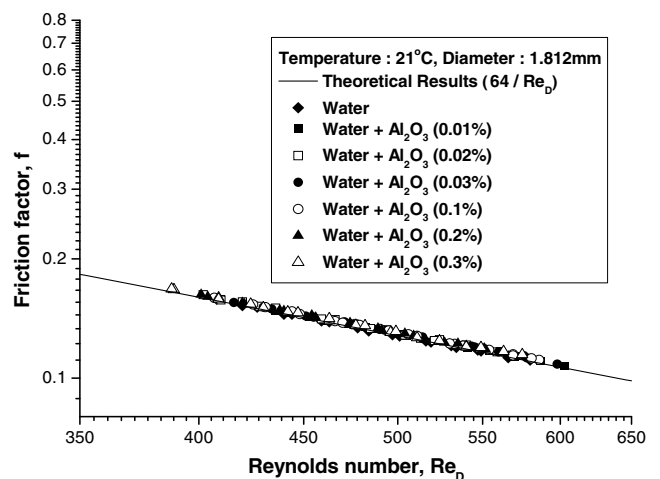


Fig. 6. The friction factor of water-based Al_2O_3 nanofluids in fully developed laminar flow.

The convective heat transfer coefficient of water-based Al_2O_3 nanofluids flowing through a circular tube for a constant heat flux is experimentally measured to understand convective heat transfer characteristics of water-based Al_2O_3 nanofluids. Fig. 7 shows the local heat transfer coefficient for water-based Al_2O_3 nanofluids as a function of position at the fixed Reynolds number of 700. The experimental results clearly show that the nanoparticles suspended in water enhance the convective heat transfer coefficient, although the volume fraction of nanoparticles is very low range from 0.01 to 0.3 vol%. The convective heat transfer coefficient of water-based Al_2O_3 nanofluids increases with volume fraction of Al_2O_3 nanoparticles as shown in Fig. 7. Especially, the convective heat transfer coefficient of water-based Al_2O_3 nanofluids is increased by 8% at 0.3 vol% under the fixed Reynolds number compared with that of pure water. In addition, the enhancement of convective heat transfer coefficient of water-based Al_2O_3 nanofluids, 8% is much higher than that of effective thermal conductivity, 1.44%, at the same volume fraction, 0.3 vol%. This trend has been observed in the previous researches [12–14,16]. Also, the experimental data of water-based Al_2O_3 nanofluids are compared with results predicted by the Shah equation which is widely used to predict the local heat transfer coefficient. In this comparison, the measured physical properties such as thermal conductivity, viscosity and heat capacity of nanofluids are applied to the Reynolds number and the Prandtl number for calculating the local heat transfer coefficient by Shah equation [26,27]. Based on the comparison, it is shown that Shah equation [26,27] fails to predict the convective heat transfer coefficient of water-based Al_2O_3 nanofluids as shown in Fig. 7. The results clearly show that the convective heat transfer of nanofluids under the fully developed condition of laminar flow is enhanced by new mechanism involved in nanofluids.

Fig. 8 shows the ratio of the convective heat transfer coefficient of water-based Al_2O_3 nanofluids to that of base fluid in fully developed laminar flow regime as a function of Reynolds number. Experimental results show the enhancement of the convective heat transfer coefficient of water-based Al_2O_3 nanofluids at the various Reynolds number compared with that of pure water. The convective heat transfer coefficient of water-based Al_2O_3 nanofluids according to the Reynolds number is about constant in fully developed laminar flow region, contrary to the previous researches which shows that the heat transfer coefficient increases with the Reynolds number at the entry region or the turbulent region [12–17]. This means that the convective heat transfer coefficient

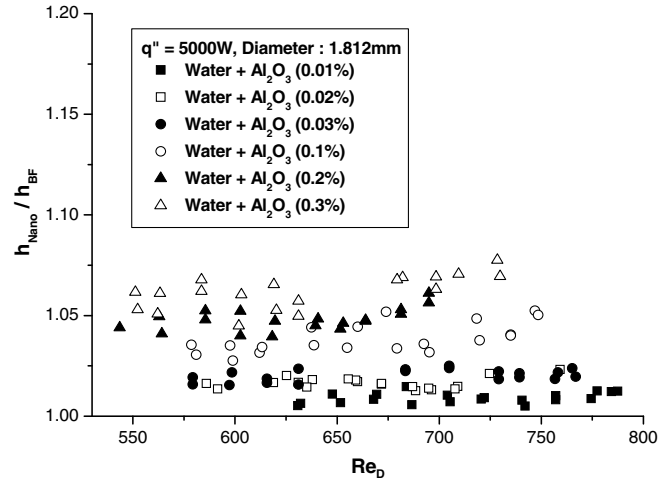


Fig. 8. The ratio of the heat transfer coefficient of water-based Al_2O_3 nanofluids to that of base fluid as a function of the Reynolds number.

of water-based Al_2O_3 nanofluids has the constant value like single-phase flow under fully developed laminar flow regime.

One of the unsolved issues in nanofluids is related to the experimental finding that the enhancement of convective heat transfer coefficient in nanofluids is higher than that of thermal conductivity. Xuan and Roetzel [20] were the first to employ the concept of thermal dispersion to explain this finding, assuming that velocity slip induces a velocity and temperature perturbation. Xuan and Li [13] suggested that the dispersion will flatten the temperature distribution, which will in turn augment the heat transfer rate between the fluid and the wall. Wen and Ding [19] numerically showed that the particle migration due to a viscosity gradient and a non-uniform shear rate leads to a higher heat transfer coefficient of nanofluids. Recently, Buongiorno [18] theoretically analyzed the effect of nanoparticle dispersion on the energy transfer of nanofluids and showed that energy transfer by nanoparticle dispersion is negligible. Instead of nanoparticle dispersion, he showed that Brownian diffusion and thermophoresis are important mechanisms for significant variations in the viscosity and thermal conductivity of nanofluids within the laminar sublayer of the turbulent boundary layer. While these mechanisms have been suggested, none has shown the flattening of velocity profiles of nanofluids as the nanoparticle concentration increases. Therefore, we have employed scale analysis and numerical solutions to show, for the first time, the flattening of velocity profile, induced from large gradients in bulk properties such as nanoparticle concentration, thermal conductivity and viscosity.

Based on the experimental results, the various effects of the effective thermal conductivities under static and dynamic conditions, energy transfer by nanoparticles' dispersion, particle migration due to viscosity gradient, non-uniform shear rate, Brownian diffusion and thermophoresis on the remarkable enhancement of the convective heat transfer coefficient are discussed to understand convective heat transfer characteristics of water-based Al_2O_3 nanofluids flowing through a circular tube.

3.1. Thermal conductivities under static and dynamic conditions

First of all, the convective heat transfer enhancement of water-based Al_2O_3 nanofluids may be attributed to the effective thermal conductivity enhancement of water-based Al_2O_3 nanofluids because the heat transfer coefficient, h is proportional to thermal conductivity, k . However, as stated above, the enhancement of convective heat transfer coefficient, 8%, is much higher than the

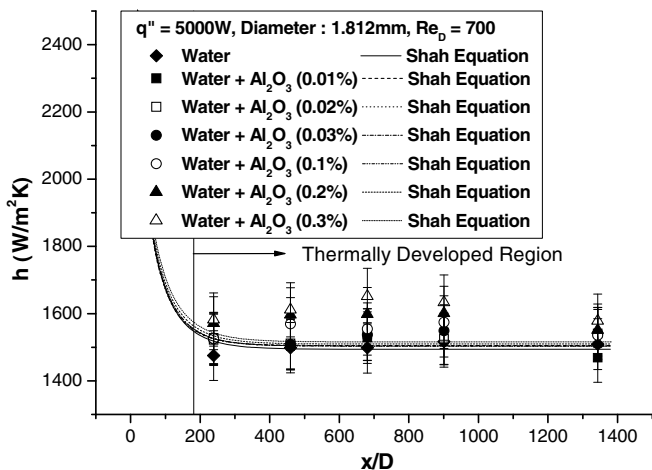


Fig. 7. The local heat transfer coefficient of water-based Al_2O_3 nanofluids in fully developed laminar flow.

increase of thermal conductivity, 1.44% at 0.3 vol%. Thus, the thermal conductivity enhancement has a small effect on the enhancement of convective heat transfer coefficient. In addition, the thermal conductivity under dynamic condition may be much higher than that under static condition. Such behavior has been observed previously by Shon and Chen [22] who studied the heat transfer of flow with micrometer or millimeter particles. Shon and Chen [22] measured the effective thermal conductivity in a rotating Couette flow apparatus at low Reynolds number but high Peclet number conditions, where the Peclet number is defined as $d_p^2 \dot{\gamma} / \alpha$ with d_p particle diameter, $\dot{\gamma}$ the shear rate and α thermal diffusivity. They show that significant enhancement of the effective thermal conductivity under dynamic condition was observed at the Peclet number over 300. However, in this study using nanometer-sized particles, the Peclet number is an order of 10^{-5} . Therefore, the effect of thermal conductivity under dynamic condition on the enhancement of heat transfer coefficient of water-based Al_2O_3 nanofluids is negligible.

3.2. Energy transfer by nanoparticle dispersion

Several researchers [12,13,20] presented that the heat transfer coefficient of nanofluids is enhanced by the dispersion of the suspension nanoparticles such as nanoparticles' random motion which is occurred by slip mechanism. Especially, Buongiorno [18] presented seven slip mechanisms and concluded that Brownian diffusion and thermophoresis are important slip mechanism in nanofluids with order of magnitude analysis. However, he approximately showed that the energy transfer by nanoparticles' dispersion is negligible with a non-dimensional analysis of energy equation. So in this paper, we try to estimate the order of the dispersion term in energy equation and then compared the order of the dispersion term with that of the other terms with this experimental case

$$\left[\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T \right] = \alpha \nabla^2 T + D_B \nabla \phi \cdot \nabla T + D_T \frac{\nabla T \cdot \nabla T}{T} \quad (12)$$

where α , $D_B = \frac{k_B T}{3\pi\mu d_p}$, $D_T = K \frac{\mu}{\rho} \phi$ are thermal diffusivity, Brownian diffusion coefficient, and thermal diffusion coefficient, respectively. Based on the results from the scale analysis, when the conduction term at the right-hand side is order of 1, the second term which indicates the dispersion term by Brownian diffusion is order of 10^{-6} , the third term which indicates the dispersion term by thermophoresis is order of 10^{-6} . Therefore, we confirm that the energy transfer by nanoparticles' dispersion due to the two important slip mechanisms which are Brownian diffusion and thermophoresis is negligible.

3.3. Particle migration

Wen and Ding [19] first numerically presented the non-uniform thermal conductivity profile resulting from particle migration induced by viscosity gradient, non-uniform shear rate and Brownian diffusion. They numerically showed that the particle migration due to the three effects could lead to a higher Nusselt number at the Peclet number, defined as $d_p^2 \dot{\gamma} / D_B$, higher than approximately 10. However, Wen and Ding [19] did not consider particle flux due to thermophoresis. So, in this paper, the respective scales of nanoparticle flux terms involved in the continuity equation for nanoparticles as given by Eq. (13) under fully developed laminar flow regime are estimated with the order of magnitude analysis

$$D_B \nabla \phi + \frac{D_T}{T} \nabla T + K_\mu \dot{\gamma} \phi^2 \frac{d_p^2}{\mu_{\text{Nano}}} \frac{d\mu_{\text{Nano}}}{d\phi} \nabla \phi + K_c d_p^2 \left[\phi^2 \nabla \dot{\gamma} + \phi \dot{\gamma} \nabla \phi \right] = 0 \quad (13)$$

where K_μ and K_c of order unity are constant related to the viscosity gradient and proportionality constant related to a non-uniform shear rate [28].

From the scale analysis, respective terms are

$$\begin{aligned} & D_B \frac{\Delta \phi}{d_{\text{tube}}} \quad , \quad \frac{D_T}{T} \frac{\Delta T}{d_{\text{tube}}} \quad , \quad K_\mu \dot{\gamma} \phi^2 \frac{d_p^2}{\mu_{\text{Nano}}} \frac{\Delta \mu_{\text{Nano}}}{d_{\text{tube}}} \\ & \text{Brownian diffusion} \quad \text{Thermophoresis} \quad \text{Viscosity gradient} \\ & K_c d_p^2 \left[\phi^2 \frac{\Delta \dot{\gamma}}{d_{\text{tube}}} + \phi \dot{\gamma} \frac{\Delta \phi}{d_{\text{tube}}} \right] \\ & \text{Non-uniform shear rate} \end{aligned}$$

Based on the experimental results, the respective scales in each term are given as follows: $d_{\text{tube}} \sim 10^{-3}$, $d_p \sim 10^{-8}$, $D_B \sim 10^{-11}$, $D_T \sim 10^{-11}$, $\phi \sim 10^{-3}$, $\Delta \phi \sim 10^{-3}$, $T \sim 10^2$, $\Delta T \sim 10$, $\mu_{\text{Nano}} \sim 10^{-3}$, $\Delta \mu_{\text{Nano}} \sim 10^{-4}$, $\dot{\gamma} \sim 10^3$, $\Delta \dot{\gamma} \sim 10^3$, $K_\mu \sim 1$ [28], $K_c \sim 1$ [28]. By using these values, the order of the respective terms are given by

$$\begin{aligned} & D_B \frac{\Delta \phi}{d_{\text{tube}}} \sim 10^{-11} \quad , \quad \frac{D_T}{T} \frac{\Delta T}{d_{\text{tube}}} \sim 10^{-9} \quad , \quad K_\mu \dot{\gamma} \phi^2 \frac{d_p^2}{\mu_{\text{Nano}}} \frac{\Delta \mu_{\text{Nano}}}{d_{\text{tube}}} \sim 10^{-17} \\ & \text{Brownian diffusion} \quad \text{Thermophoresis} \quad \text{Viscosity gradient} \\ & K_c d_p^2 \left[\phi^2 \frac{\Delta \dot{\gamma}}{d_{\text{tube}}} + \phi \dot{\gamma} \frac{\Delta \phi}{d_{\text{tube}}} \right] \sim 10^{-16} \\ & \text{Non-uniform shear rate} \end{aligned}$$

From order of magnitude analysis we find that, of these four terms, thermophoresis as well as Brownian diffusion is major effect on particle migration and the effect of viscosity gradient and non-uniform shear rate on particle migration can be negligible. Therefore, under fully developed laminar flow regime, nanoparticle continuity equation, momentum equation, and energy equation for nanofluids [18] can be expressed as

$$\text{Nanoparticle continuity equation: } \nabla \cdot \left[D_B \nabla \phi + D_T \frac{\nabla T}{T} \right] = 0 \quad (14)$$

$$\text{Momentum equation: } \nabla p + \nabla \cdot \mu_{\text{Nano}} \nabla u = 0 \quad (15)$$

$$\text{Energy equation: } \rho c u \nabla T = \nabla \cdot k_{\text{Nano}} \nabla T \quad (16)$$

Particle movement into centerline of a tube due to Brownian diffusion and thermophoresis in Eq. (14) significantly increases the viscosity and thermal conductivity of nanofluids near the centerline. Also, as a consequence of the viscosity increment of nanofluids near the centerline, velocity profile predicted by Eq. (15) is flattened as shown in Fig. 9. In addition, the flattened velocity profile decrease the difference between the tube wall temperature and bulk mean temperature of nanofluids under the constant heat flux as given by the following equation:

$$h = \frac{q''}{T_s - T_m} \quad \text{where } T_m = \frac{1}{u_m A_C} \int_{A_C} u T dA_C \quad (17)$$

With the decrease of the difference between tube wall temperature and bulk mean temperature of nanofluids and a little thermal conductivity enhancement of nanofluids the convective heat transfer coefficient of Al_2O_3 water-nanofluids is theoretically estimated to be enhanced by 5% at 0.3 vol%. These theoretical results can explain the enhancement 8% at 0.3 vol% experimentally measured in this paper.

4. Conclusion

This study is to investigate flow and convective heat transfer characteristics of water-based Al_2O_3 nanofluids flowing through a circular tube of 1.812 mm inner diameter with the constant heat flux in fully developed laminar regime. For the purpose, water-based Al_2O_3 nanofluids with various volume fractions ranging from 0.01% to 0.3% are manufactured by the two-step

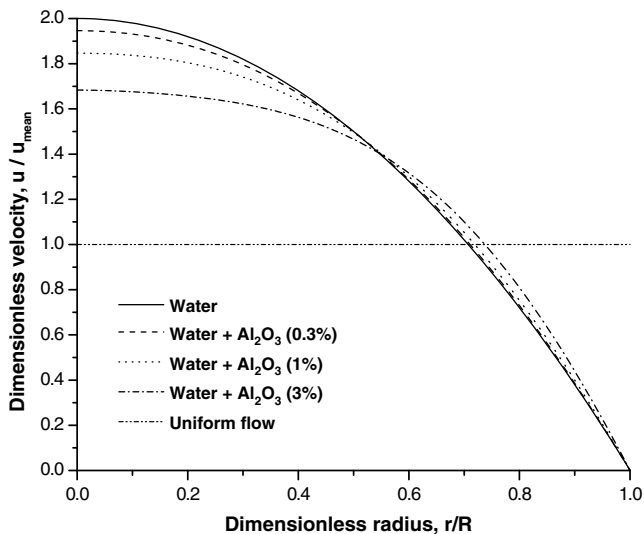


Fig. 9. Velocity profile for fully developed laminar flow in a circular tube.

method which indicates that base fluid including nanoparticles is sonicated in an ultrasonic bath. In addition, physical properties of water-based Al_2O_3 nanofluids such as the viscosity, the density, the thermal conductivity and the heat capacity are measured. To investigate flow and convective heat transfer characteristics, we measure the pressure drop and convective heat transfer coefficient through the circular tube.

Based on the experimental results, it is shown that the Darcy friction factor of water-based Al_2O_3 nanofluids experimentally measured in this paper has a good agreement with theoretical results from the friction factor correlation for the single-phase flow ($f = 64/Re_D$). In addition we clearly present that the nanoparticles suspended in water enhance the convective heat transfer coefficient in the thermally fully developed regime, despite low volume fraction between 0.01 and 0.3 vol%. Especially, the heat transfer coefficient of water-based Al_2O_3 nanofluids is increased by 8% at 0.3 vol% under the fixed Reynolds number compared with that of pure water and the enhancement of the heat transfer coefficient is larger than that of the effective thermal conductivity at the same volume concentration. Also, the convective heat transfer coefficient of water-based Al_2O_3 nanofluids is increased with volume fraction of Al_2O_3 nanoparticles, but cannot be predicted by Shah equation [26,27]. Finally, the various effects of thermal conductivities under static and dynamic conditions, energy transfer by nanoparticles dispersion, particle migration due to viscosity gradient, non-uniform shear rate, Brownian diffusion and thermophoresis on remarkable enhancement of the convective heat transfer coefficient are discussed to understand convective heat transfer characteristics of water-based Al_2O_3 nanofluids flowing through a circular tube. Based on scale analysis and numerical solutions, we show, for the first time, the flattened velocity profile due to particle migration induced by Brownian diffusion and thermophoresis. It is shown that the flattening of velocity profile is a possible mechanism of the convective heat transfer enhancement, which cannot be explained by an increase in the thermal conductivity of nanofluids alone.

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